Arab Academy for Science &Technology and Maritime Transport (AASTMT)
College of Computing and Information Technology (CCIT)
Theory of Computation CS311 – Spring 2014
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Turing Machines Examples

Example 1:

We know $L=\{0^n1^n0^n \mid n \geq 0\}$ is not a CFL (pumping lemma)

Can we show L is decidable? Construct a decider M such that L(M) = L A decider is a TM that always halts (in q_{acc} or q_{rej}) and is guaranteed not to go into an infinite loop for any input

Input: 000001111100000

Idea: Mark off matching 0s, 1s, and 0s with Xs (left end marked with blank)

000001111100000

_00001111100000

_0000X111100000

_0000X1111X0000

_X000X1111X0000

....

Idea for a Decider for $\{0^n1^n0^n \mid n \geq 0\}$

General Idea: Match each 0 with a 1 and a 0 following the 1.

1 Implementation Level Description of a Decider for L:

On input w:

1. If first symbol = blank, ACCEPT

2. If first symbol = 1, REJECT

3. If first symbol = 0, Write a blank to mark left end of tape

    a. If current symbol is 0 or X, skip until it is 1. REJECT if blank.

    b. Write X over 1. Skip 1's/X's until you see 0. REJECT if blank.

    c. Write X over 0. Move back to left end of tape.

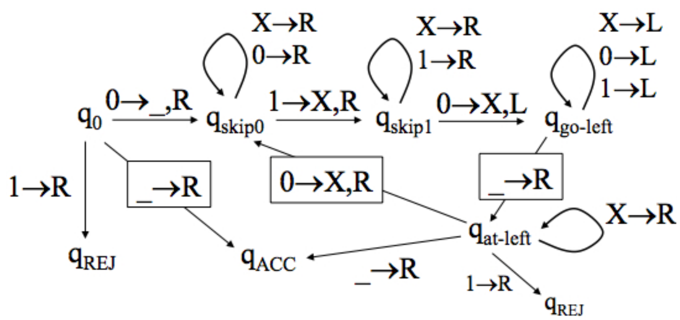
4. At left end: Skip X's until:

a. You see 0: Write X over 0 and GOTO 3a

b. You see 1: REJECT

c. You see a blank space: ACCEPT

State Diagram



Note: Some transitions to q_{REJ} (e.g., from q_{skip0}) are not shown to avoid clutter

Try running the decider on:

010, 001100, ...ACCEPT

0, 000, 0100, ...REJECT

What about 010010?

The decider accepts incorrect strings:

010010, 010001100ACCEPT!!!

Accepts $(0^n 1^n 0^n)^*$

Need to fix it...How??

A Simple Fix (to the Decider)

Scan initially to make sure string is of the form 0*1*0*

On input w:

1. If first symbol = blank, ACCEPT

2. If first symbol = 1, REJECT

3. If first symbol = 0: if w is not in 00*11*00*, REJECT; else,

Write a blank to mark left end of tape

a. If current symbol is 0 or X, skip until it is 1. REJECT if blank.

b. Write X over 1. Skip 1's/X's until you see 0. REJECT if blank.

c. Write X over 0. Move back to left end of tape.

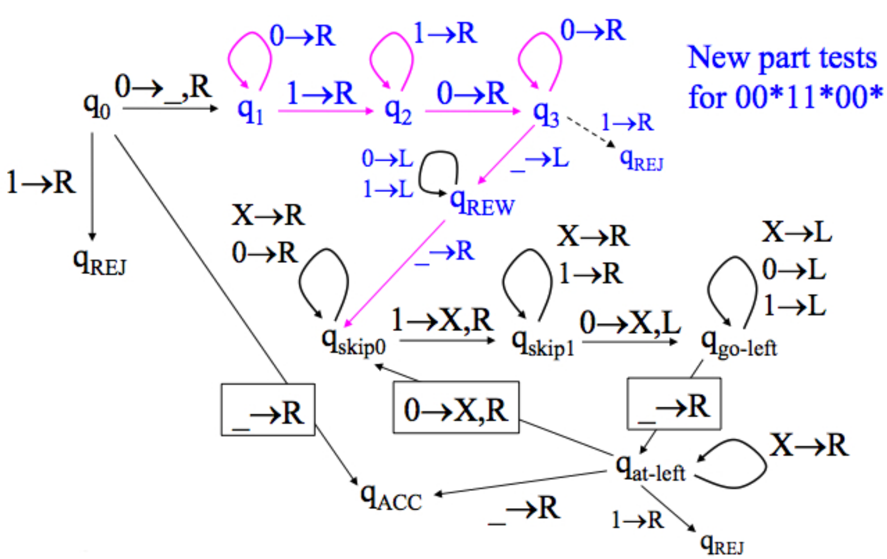
4. At left end: Skip X's until:

a. You see 0: Write X over 0 and GOTO 3a

b. You see 1: REJECT

c. You see a blank space: ACCEPT

The Decider TM for L in all its glory

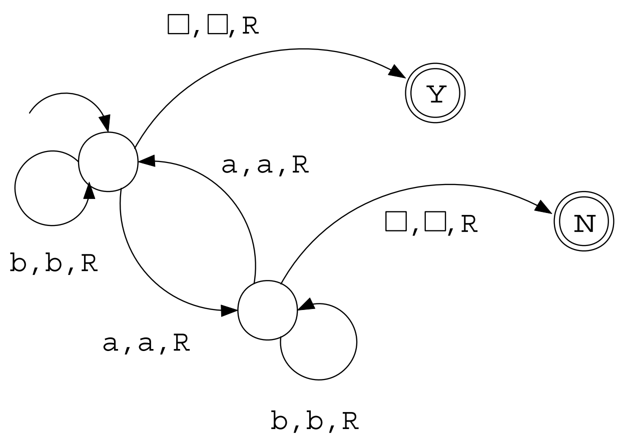


Example 2:

Design a Turing machine which returns whether an input ranging over $\{a, b\}^*$ has an even number of a's.

State	Read	Write	Next State	Move
q_0	a	a	q_1	R
q_0	b	b	q_0	R
q_0	$\square$	$\square$	q_Y	S
q_1	a	a	q_0	R
q_1	b	b	q_1	R
q_1	$\square$	$\square$	q_N	S

Graphically, this can be expressed as:



Example 3:

Here, a TM M_3 is doing some elementary arithmetic. It decides the language C = $\{a^i b^j c^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$.

M_3 = "On input string w:

1. Scan the input from left to right to determine whether it is a member of $a^+b^+c^+$ and reject if it isn't.

2. Return the head to the left-hand end of the tape.

3. Cross off an a and scan to the right until a b occurs. Shuttle between the b's and the c's, crossing off one of each until all b's are gone. If all c's have been crossed off and some b's remain, reject .

4. Restore the crossed off b's and repeat stage 3 if there is another a to cross off. If all a's have been crossed off, determine whether all c's also have been crossed off. If yes, accept; otherwise, reject ."

Tracing of w = aabbbcccccc :

xabbbcccccc

xayyyzzzccc

xabbbzzzccc

xxyyyzzzzzz

Example 4:

A TM to add 1 to a binary number (with a 0 in front)

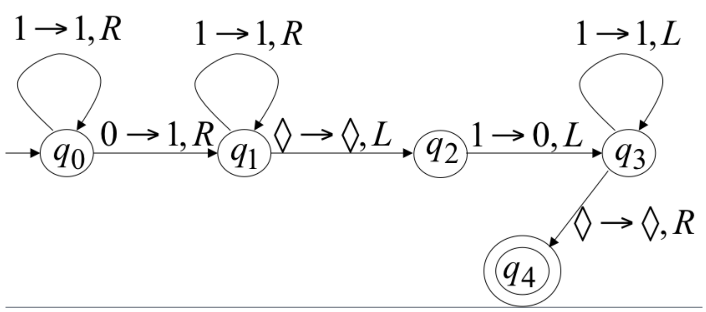
M = “On input w

1. Go to the right end of the input string
2. Move left as long as a 1 is seen, changing it to a 0.
3. Change the 0 to a 1, and halt.”

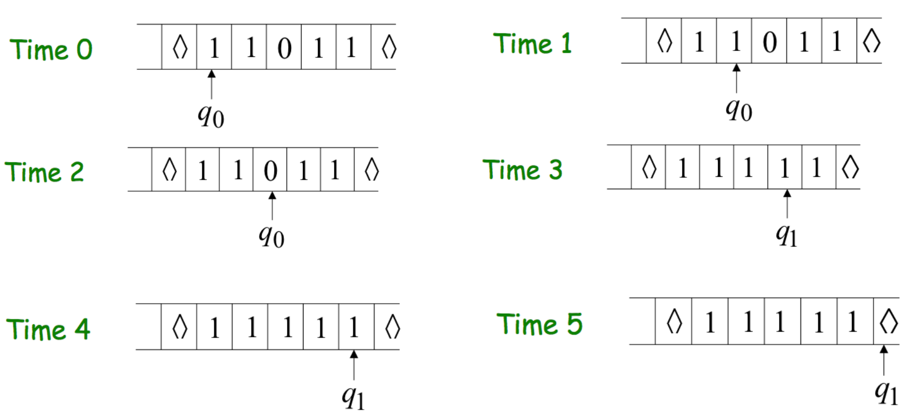
For example, to add 1 to w = 0110011 Change all the ending 1’s to 0’s ⇒ 0110000 Change the next 0 to a 1 ⇒ 0110100

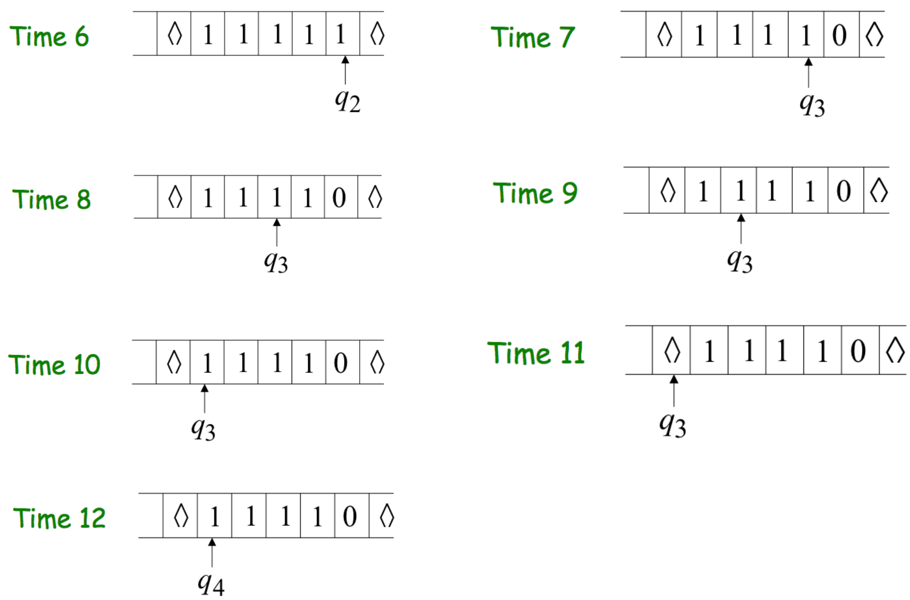
Example 5:

A TM to add two numbers: f(x, y) = x + y



when x = 11, and y = 11, the computation proceeds as follows:





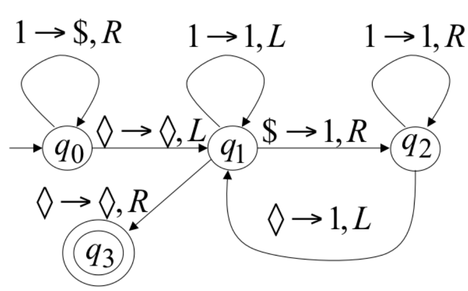
Example 6:

A TM to compute: f(x) = 2x.

The TM takes x as unary input, and write in the tape xx as unary

Pseudo-code:

- Replace every 1 with $
- Repeat:
    - Find rightmost $, replace it with 1
    - Go to right end, insert 1
- Until no more $ remain



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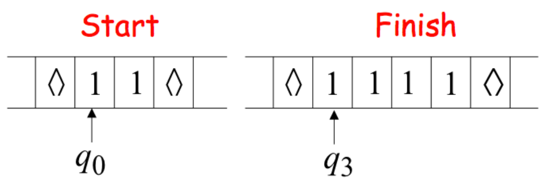


Example 7:

A TM to compute:

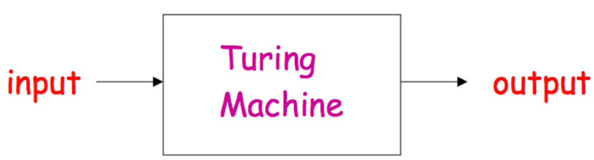
$$f(x, y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{if } x \leq y \end{cases}$$

The TM takes x0y as input, and writes in the tape 1 or 0

Pseudo-code:

- Repeat
  - Match a 1 from x with a 1 from y
- Until all of x or y is matched
- If a 1 from x is not matched
  - Erase tape, write 1
- else
  - Erase tape, write 0

Combining Turing Machines:



$$f(x, y) = \begin{cases} x + y & \text{if } x > y \\ 0 & \text{if } x \leq y \end{cases}$$