



Arab Academy for Science & Technology and Maritime Transport (AASTMT)

College of Computing and Information Technology (CCIT)

Theory of Computation CS311 – Spring 2014

Dr. Manal Helal

Eng. Nada Mahmoud

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1. For $\Sigma : \{a, b\}$, design a Turing machine that accepts:

$$L = \{a^n b^n : n \geq 1\}$$

Intuitively, we solve the problem in the following fashion. starting at the Leftmost a, we check it off by replacing it with some symbol, say x. We then let the read-write head travel right to find the leftmost b, which in turn is checked off by replacing it with another symbol, say y. After that, we go left again to the leftmost a, replace it with an x, then move to the leftmost b and replace it with y, and so on. Traveling back and forth this way we match each a with a corresponding b. If after some time no a's or b's remain, then the string must be in L.

working out the details, we arrive at a complete solution for which:

$$Q: \{q_0, q_1, q_2, q_3, q_4\}$$

$$F: \{q_4\}$$

$$\Sigma: \{a, b\},$$

$$\Gamma: \{a, b, x, y, \square\}$$

The transitions can be broken into several parts. The set

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, b) = (q_2, y, R)$$

$$\delta(q_2, b) = (q_2, b, L)$$

Replaces the leftmost a with an x, then causes the read-write head to travel right to the first b, replacing it with a y. when the y is written, the machine enters state q_2 , indicating that an a has been successfully paired with a b.

The next set of transitions reverses the direction until an x is encountered, repositions the read-write head over the leftmost a, and returns control to the initial state.

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

We are now back in the initial state q_0 , ready to deal with the next a and b. After one pass through this part of the computation, the machine will have carried out the partial computation:

$$q_0 a a \dots a b b \dots b \vdash x q_1 a \dots a y b \dots b$$

So that a single a has been matched with a single b. After two passes, we will have completed the partial computation

$$q_0 a a \dots a b b \dots b \vdash^* x x q_1 \dots a y y \dots b$$

And so on, indicating that the matching process is being carried out properly.

When the input is a string $a^n b^n$, the rewriting continues this way, stopping only when there are no more a's to be erased. When looking for the leftmost a, the read-write head travels left with the machine in state q_2 . When an x is encountered, the direction is reversed to get the a. But now, instead of finding an a it will find a y. To terminate, a final check is made to see if all a's and b's have been replaced (to detect input where an a follows a b). This can be done by:

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R),$$

$$\delta(q_3, \square) = (q_4, \square, R).$$

If we input a string not in the language, the computation will halt in a non-final state. For example, if we give the machine a string $a^n b^m$, with $n > m$, the machine will eventually encounter a blank in state q_1 . It will halt because no transition is specified for this case. Other input not in the language will also lead to a non-final halting state.

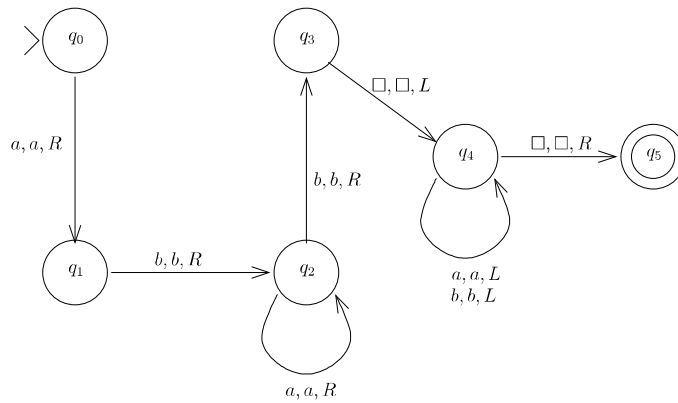
The particular input aabb gives the following successive instantaneous descriptions:

$$\begin{aligned} q_0 a a b b \vdash & \quad x q_1 a b b \vdash x a q_1 b b \vdash x q_2 a y b \vdash \\ & q_2 x a y b \vdash x q_0 a y b \vdash x x q_1 y b \vdash \\ & x x y q_1 b \vdash x x q_2 y y \vdash x q_2 x y y \vdash \\ & x x q_0 y y \vdash x x y q_3 y \vdash x x y y q_3 \square \vdash \\ & x x y y \square q_4 \square. \end{aligned}$$

At this point the Turing machine halts in a final state, so the string aabb is accepted.

Trace with "ab", and "aaabbb"

2. Consider the following Turing machine M:



- Give the computation path trace for initial configuration q_0abab .
- Give the computation path trace for initial configuration q_0abba .
- Give a regular expression for $L(M)$.

Solutions:

- Give the computation path trace for initial configuration q_0abab .

$q_0abab \vdash aq_1bab \vdash abq_2ab \vdash abaq_2b \vdash ababq_3 \square \vdash abaq_4b \vdash$

$abq_4ab \vdash aq_4bab \vdash q_4abab \vdash q_4 \square abab \vdash q_5abab$

- Give the computation path trace for initial configuration q_0abba .

$q_0abba \vdash aq_1bba \vdash abq_2ba \vdash abbq_3a$

- Give a regular expression for $L(M)$.

aba^*b