



Arab Academy for Science & Technology and Maritime Transport (AASTMT)

College of Computing and Information Technology (CCIT)

Theory of Computation CS311 – Spring 2014

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Use the pumping lemma to show that the following languages are not context free.

- a. $\{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$
- b. $\{w \# t \mid w \text{ is a substring of } t, \text{ where } w, t \in \{a, b\}^*\}$

Solution:

(a) Let $B = \{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$. Let p be the pumping length given by the pumping lemma. Let $s = 0^p \# 0^{2p} \# 0^{3p}$. We show that $s = uvxyz$ cannot be pumped. Neither v nor y can contain $\#$, otherwise uv^2xy^2z contains more than two $\#$ s. Therefore, if we divide s into three segments by $\#$'s: 0^p , 0^{2p} , and 0^{3p} , at least one of the segments is not contained within either v or y . Hence uv^2xy^2z is not in B because the 1 : 2 : 3 length ratio of the segments is not maintained.

(b) Let $C = \{w \# t \mid w \text{ is a substring of } t, \text{ where } w, t \in \{a, b\}^*\}$. Let p be the pumping length given by the pumping lemma. Let $s = a^p b^p \# a^p b^p$. We show that the string $s = uvxyz$ cannot be pumped.

Neither v nor y can contain $\#$, otherwise uv^0xy^0z does not contain $\#$ and therefore is not in C . If both v and y occur on the left-hand side of the $\#$, the string uv^2xy^2z cannot be in C because it is longer on the left-hand side of the $\#$. Similarly, if both strings occur on the right-hand side of the $\#$, the string uv^0xy^0z cannot be in C because it is again longer on the left-hand side of the $\#$. If one of v and y is empty (both cannot be empty), treat them as if both occurred on the same side of the $\#$ as above.

The only remaining case is where both v and y are nonempty and straddle the $\#$. But then v consists of b 's and y consists of a 's because of the third pumping lemma condition $|vxy| \leq p$. Hence, uv^2xy^2z contains more b 's on the left-hand side of the $\#$, so it cannot be a member of C .