

The objective of this problem is for the student to pay close attention to the exact formulation of the pumping lemma.

- a. The minimum pumping length is 4. The string $s = 000 \in 0001^*$ cannot be pumped, and any string in 0001^* of length 4 or more contains a 1 and hence can be pumped by dividing it so that $x = 0$, $y = 1$, and z is the rest.
- b. The minimum pumping length is 1. The string ϵ cannot be pumped because of condition 2 in the pumping lemma and hence 0 is not a pumping length. Any string in 0^*1^* of length 1 or more contains a 0 or a 1 and hence can be pumped by dividing it so that $x = \epsilon$, $y = 0$ or 1 , and z is the rest, so 1 is a pumping length.
- c. The minimum pumping length is 1. The pumping length cannot be 0, as in part (b). Any string in $(01)^*$ of length 1 or more contains 01 and hence can be pumped by dividing it so that $x = \epsilon$, $y = 01$, and z is the rest.
- d. The minimum pumping length is 3. The string 01 cannot be pumped, so 2 is not a pumping length. All strings in the language of length 3 or more can be pumped, (note that there aren't any such string, so the statement is true vacuously) so 3 is a pumping length.
- e. The minimum pumping length is 1. String ϵ cannot be pumped because of condition 2 in the pumping lemma and hence 0 is not a pumping length. The language has no strings of length 1 or more so 1 is a pumping length. (the conditions hold vacuously).