



Arab Academy for Science & Technology and Maritime Transport (AASTMT)

College of Computing and Information Technology (CCIT)

Theory of Computation CS311 – Spring 2014

Dr. Manal Helal

Eng. Nada Mahmoud

Section 8 – 26th of April 2014

Give the Context Free Grammar (CFG) for the following languages:

1. $\{a^i b^j \mid i, j \geq 1 \text{ and } |i-j| \leq 3\}$

$S \rightarrow aSb \mid aXb$ (generates $a^{\min(i,j)}$ or $b^{\min(i,j)}$)

$X \rightarrow aaa \mid aa \mid a \mid \epsilon \mid b \mid bb \mid bbb$

2. $\{a^i b^{2j} c^k d^l \mid i, j, k \geq 1 \text{ and } k \geq 2i\}$

$S \rightarrow Sd \mid X$ (generates $k-2i$ ds)

$X \rightarrow aXdd \mid aYdd$

$Y \rightarrow bbYc \mid bbc$

3. $\{x \mid x \in \{a, b\}^*, abb, aab, \text{ are substrings of } x\}$

X generates $abb \dots aab \dots$ Y generates $aab \dots abb$

$S \rightarrow X \mid Y$

$X \rightarrow Xa \mid Xb \mid Pb$

$P \rightarrow Qa$

$Q \rightarrow Ra$

$R \rightarrow Ra | Rb | Tb$

$T \rightarrow Ub$

$U \rightarrow Va$

$Y \rightarrow Ya | Yb | Ab$

$A \rightarrow Bb$

$B \rightarrow Ca$

$C \rightarrow Ca | Cb | Db$

$D \rightarrow Ea$

$E \rightarrow Va$

$V \rightarrow Va | Vb | \epsilon$

4. $\{a^i b^j \mid i, j \geq 0 \text{ and } i \neq j\}$

$S \rightarrow aSb \mid A \mid B$

$A \rightarrow Aa \mid a$

$B \rightarrow Bb \mid b$

5. $\{xycy \mid x, y \in \{a, b\}^*, x^R \text{ is a prefix of } y\}$

$y = x^R u$ such that $u \in \{a, b\}^*$

string = xcx^Ru

$S \rightarrow Sa \mid Sb \mid X$

$X \rightarrow aXa \mid bXb \mid c$

6. $\{x_1 x_2 cy \mid x_1, x_2, y \in \{a, b\}^*, (\exists x \in \{a, b\}^*) (x_1 x x_2 = y^R)\}$

$y = x_2^R x^R x_1^R = x_2^R u x_1^R$ such that $u \in \{a, b\}^*$,

String = $x_1 x_2 c x_2^R u x_1^R$

$S \rightarrow aSa | bSb | X$ (generates $x_1 \dots x_1^R$)

$X \rightarrow Xa | Xb | Y$ (generates u)

$Y \rightarrow aYa | bYb | c$ (generates $x_2 c x_2^R$)