CS311: Computational Theory

Lecture 7: CONTEXT-FREE GRAMMARS – Ch 2 (Cont'd)

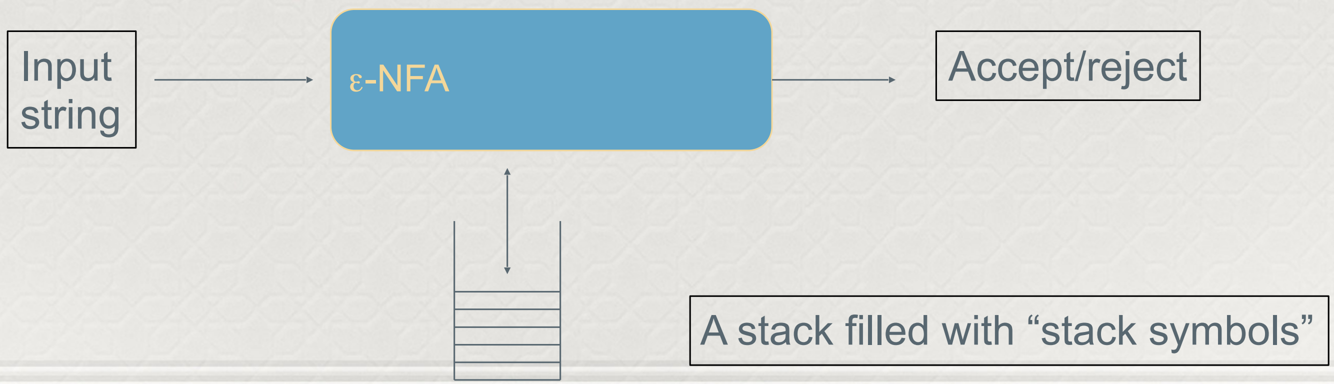
Dr. Manal Helal, Spring 2014.

Lecture Learning Objectives

1. *Express Context Free Grammar Languages using Push-Down Automaton (PDA)*

PDA - the automata for CFLs

- What is?
  - FA to Reg Lang,   PDA is to CFL
- PDA == [ε -NFA + "a stack"]
- Why a stack?



Pushdown Automata - Definition

- *A PDA P := (Q,∑,Γ, δ,q_0,Z_0,F):*
  - *Q:* *states of the ε-NFA*
  - *∑:* *input alphabet*
  - *Γ :* *stack symbols*
  - *δ:transition function*
  - *q_0:* *start state*
  - *Z_0:* *Initial stack top symbol*
  - *F:* *Final/accepting states*

old state Stack top input symb. new state(s) new Stack top(s)

δ : Q x Γ x ∑ => Q x Γ

δ : The Transition Function

δ(q,a,X) = {(p,Y), …}

1. *state transition from q to p*
2. *a is the next input symbol*
3. *X is the current stack top symbol*
4. *Y is the replacement for X; it is in Γ* (a string of stack symbols)*
   1. *Set Y = ε for: Pop(X)*
   2. *If Y=X: stack top is unchanged*
   3. *If $Y=Z_1Z_2\ldots Z_k$: X is popped and is replaced by Y in reverse order (i.e., Z_1 will be the new stack top)*

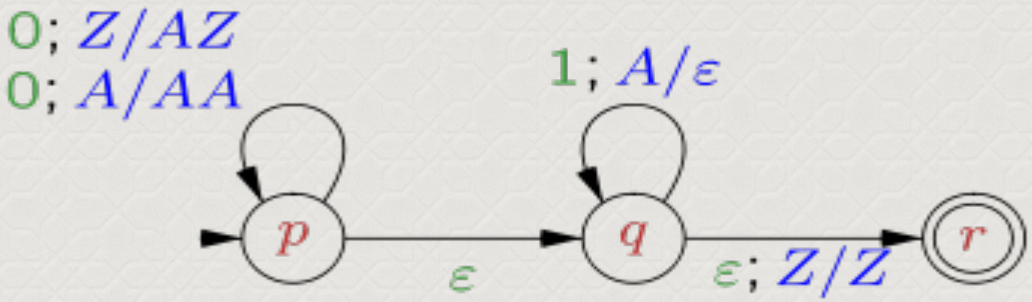
Non-determinism

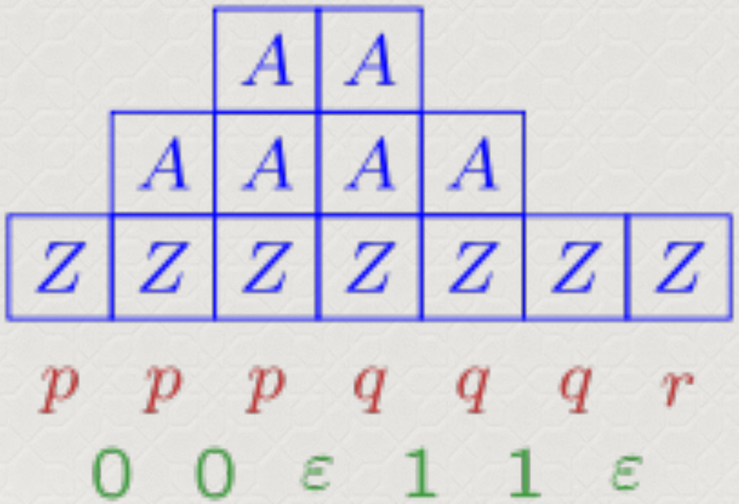
	Y = ?	Action
i)	Y=ε	Pop(X)
ii)	Y=X	Pop(X) Push(X)
iii)	$Y=Z_1Z_2..Z_k$	Pop(X) Push(Z_k) Push(Z_{k-1}) … Push(Z_2) Push(Z_1)

Example 1: PDA for $\{0^n1^n \mid n \geq 1\}$

- $P := (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$, where
- States Q: {p, q, r}
- input alphabet Σ: {0, 1}
- stack alphabet Γ: {A, Z}
- start state q_0: p
- start stack symbol Z_0: Z
- accepting states F: {r}
- δ: (p, 0, Z) => (p, AZ), (p, 0, A) => (p, AA),
  (p, ε, Z) => (q, Z), (p, ε, A) => (q, A),
  (q, 1, A) => (q, ε), and (q, ε, Z) => (r, Z)

Understanding the Computation





$$
\begin{array}{llrrl}
(p, & 0011, & Z) & \vdash \\
(p, & 011, & AZ) & \vdash \\
(p, & 11, & AAZ) & \vdash \\
(q, & 11, & AAZ) & \vdash \\
(q, & 1, & AZ) & \vdash \\
(q, & \varepsilon, & Z) & \vdash \\
(r, & \varepsilon, & Z) &
\end{array}
$$

Example 2: L_{wwr}

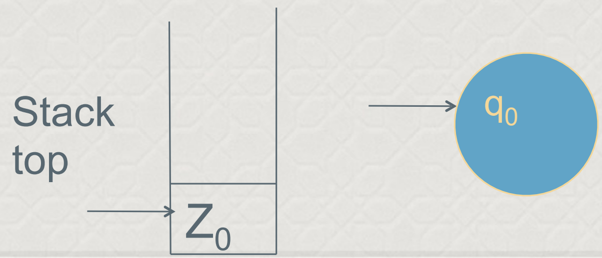
Let $L_{wwr} = \{ww^R \mid w \text{ is in } (0+1)^*\}$

- CFG for L_{wwr} : $S ==> 0S0 \mid 1S1 \mid \varepsilon$
- PDA for L_{wwr} :
- $P := (Q, \sum, \Gamma, \delta, q_0, Z_0, F)$

$= (\{q_0, q_1, q_2\}, \{0,1\}, \{0,1,Z_0\}, \delta, q_0, Z_0, \{q_2\})$

PDA for L_{wwr}

Initial state of the PDA:



1. $\delta(q_0, 0, Z_0) = \{(q_0, 0Z_0)\}$
2. $\delta(q_0, 1, Z_0) = \{(q_0, 1Z_0)\}$

First symbol push on stack

1. $\delta(q_0, 0, 0) = \{(q_0, 00)\}$
2. $\delta(q_0, 0, 1) = \{(q_0, 01)\}$
3. $\delta(q_0, 1, 0) = \{(q_0, 10)\}$
4. $\delta(q_0, 1, 1) = \{(q_0, 11)\}$

Grow the stack by pushing new symbols on top of old (w-part)

5. $\delta(q_0, \varepsilon, 0) = \{(q_1, 0)\}$
6. $\delta(q_0, \varepsilon, 1) = \{(q_1, 1)\}$
7. $\delta(q_0, \varepsilon, Z_0) = \{(q_1, Z_0)\}$

Switch to popping mode (boundary between w and w^R)

8. $\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$
9. $\delta(q_1, 1, 1) = \{(q_1, \varepsilon)\}$

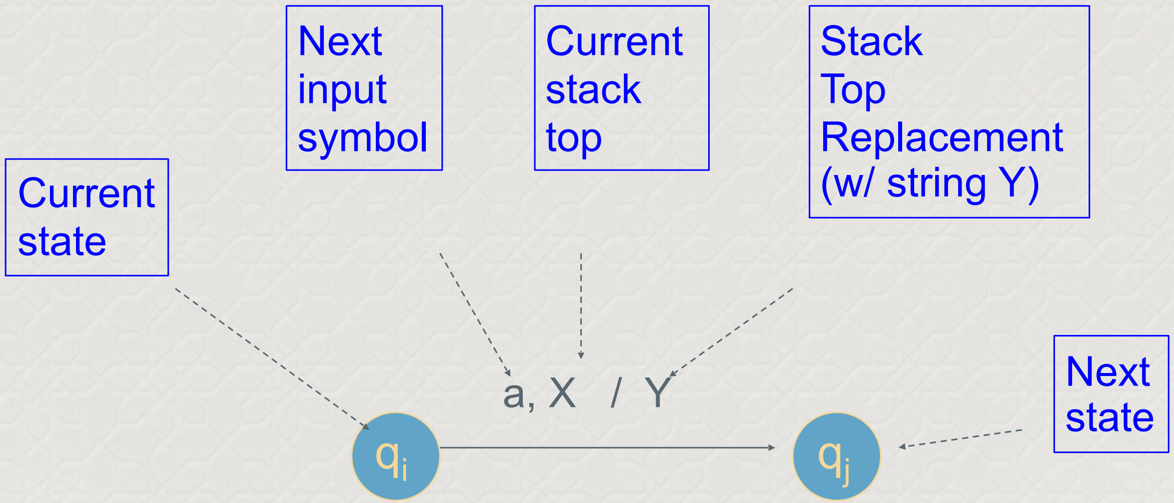
Shrink the stack by popping matching symbols (w^R-part)

10. $\delta(q_1, \varepsilon, Z_0) = \{(q_2, Z_0)\}$

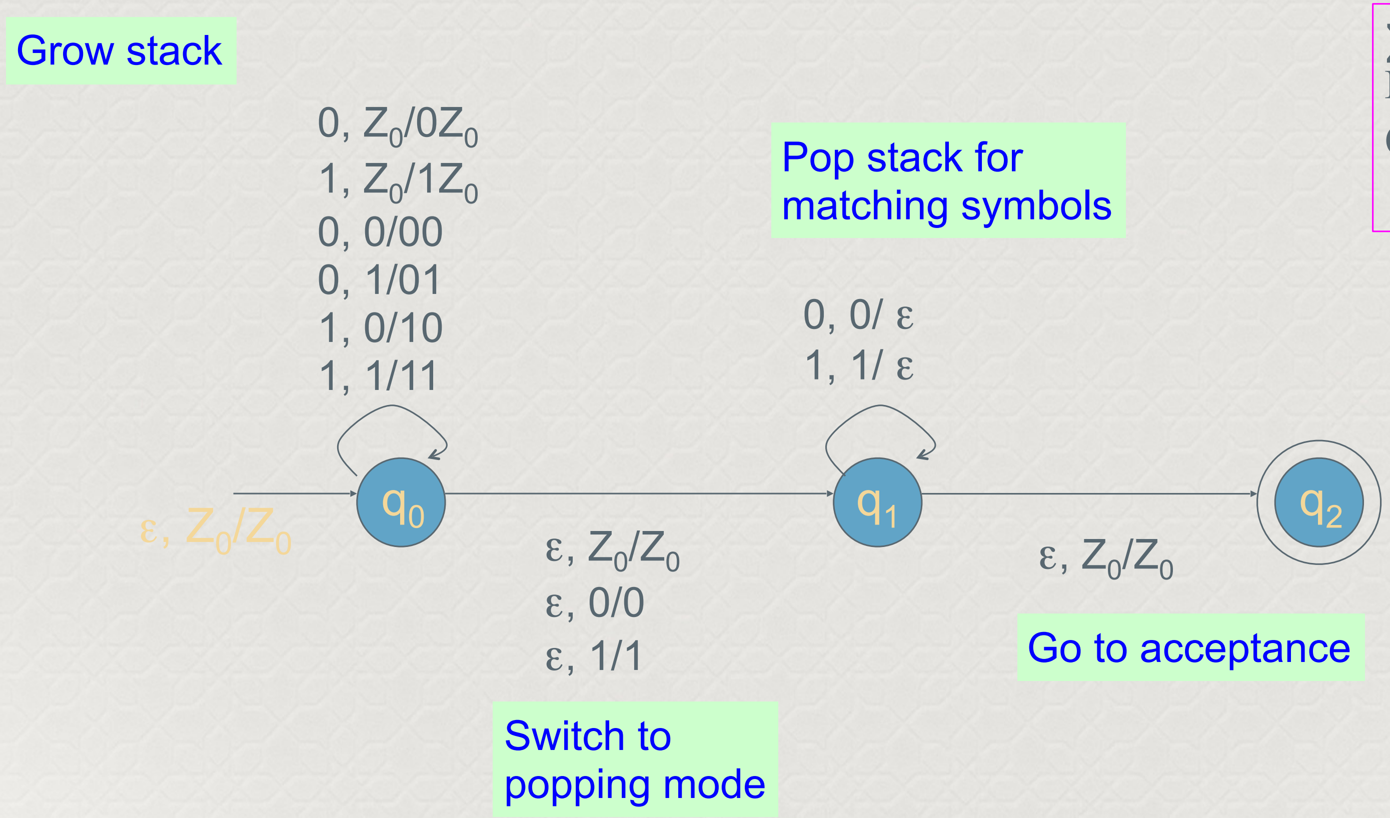
Enter acceptance state

PDA as a state diagram

$\delta(q_i, a, X) = \{(q_j, Y)\}$



PDA for L_{wwr}: Transition Diagram



$\Sigma = \{0, 1\}$
$\Gamma = \{Z_0, 0, 1\}$
$Q = \{q_0, q_1, q_2\}$

This would be a non-deterministic PDA

How does the PDA for L_{wwr} work on input "1111"?

All moves made by the non-deterministic PDA

$(q_0,1111,Z_0)$ → $(q_0,111,1Z_0)$ → $(q_0,11,11Z_0)$ → $(q_0,1,111Z_0)$ → $(q_0,\varepsilon,1111Z_0)$ → $(q_1,\varepsilon,1111Z_0)$

Path dies…

$(q_0,1111,Z_0)$ → $(q_1,1111,Z_0)$ → Path dies…

$(q_0,111,1Z_0)$ → $(q_1,111,1Z_0)$ → Path dies…

$(q_0,11,11Z_0)$ → $(q_1,11,11Z_0)$ → $(q_1,1,1Z_0)$ → (q_1,ε,Z_0) → (q_2,ε,Z_0)

$(q_0,1,111Z_0)$ → $(q_1,1,111Z_0)$ → $(q_1,\varepsilon,11Z_0)$

Path dies…

Acceptance by final state:

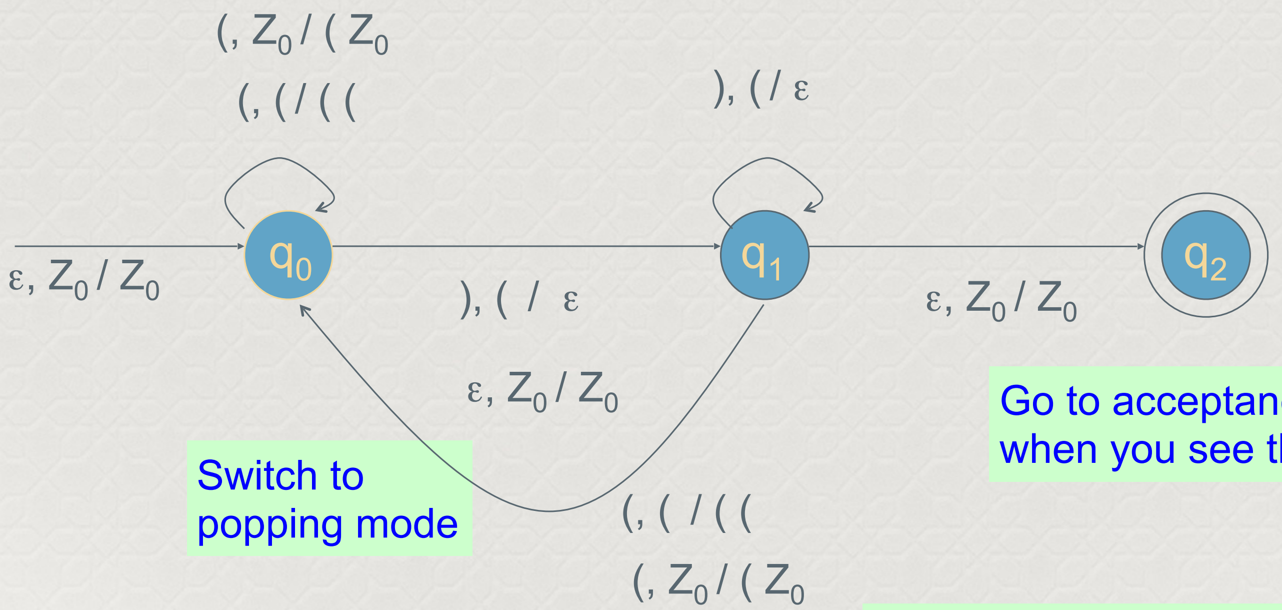
= empty input
AND
final state

Example 3: language of balanced paranthesis

Pop stack for matching symbols

$\sum = \{ (,) \}$
$\Gamma = \{Z_0, (\}$
$Q = \{q_0, q_1, q_2\}$

Grow stack

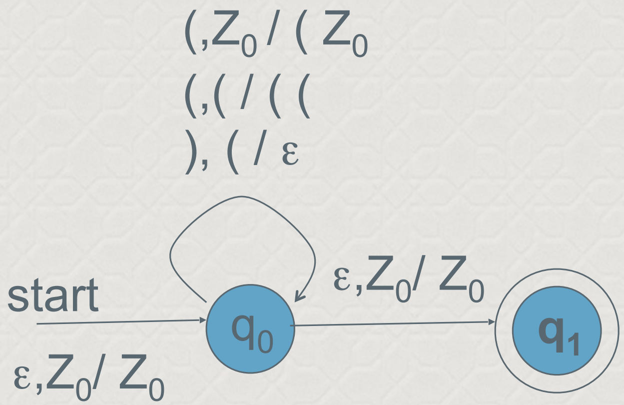


Go to acceptance (by final state) when you see the stack bottom symbol

Switch to popping mode

To allow adjacent blocks of nested paranthesis

Example 2: language of balanced paranthesis (another design)



$\sum$ = { (,) }
Γ= {Z_0, (}
Q = {q_0,q_1}

PDA's Instantaneous Description (ID)

A PDA has a configuration at any given instance: (q,w,y)

- *q - current state*
- *w - remainder of the input (i.e., unconsumed part)*
- *y - current stack contents as a string from top to bottom of stack*

If δ(q,a, X)={(p, A)} is a transition, then the following are also true:

- *(q, a, X) |--- (p,ε,A)*
- *(q, aw, XB) |--- (p,w,AB)*

|--- sign is called a "turnstile notation" and represents one move

|--- sign represents a sequence of moves*