COM2031 Advanced Algorithms, Autumn Semester 2019

Lab 8: Applications of Network Flow

Purpose of the lab

Your task is to solve the following problems using the Ford Fulkerson algorithm for network flows. You should do this by transforming the problem into a flow problem, apply Ford Fulkerson to solve it, and then transform the result back to a solution to the original problem.

You are provided with an implementation of the Ford Fulkerson algorithm, or you can use your own one that you developed in Lab 7 if you prefer.

1. Bipartite Matching

1a) The following table gives a list of 11 musicians and the 11 instruments that they play.

Alice	Guitar, Oboe, Drums, Saxophone
Bob	Violin, Cello, Guitar
Chris	Piano, Drums, Saxophone
Donna	Flute, Oboe
Eric	Bass, Clarinet, Cello
Freddy	Saxophone, Flute, Guitar
Gabriella	Cello, Guitar, Piano, Bassoon
Henry	Oboe, Saxophone
Ian	Bassoon, Flute, Clarinet
Jess	Oboe, Guitar, Drums
Kelly	Guitar, Drums, Flute

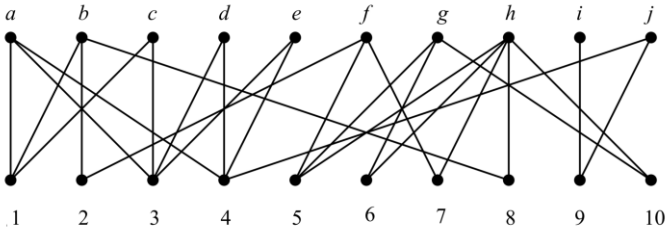
Find a maximal bipartite matching between musicians and instruments, and see whether or not it is perfect.

Slides 19-21 of Week 8 lectures cover the following: If the maximal matching is not perfect then Hall's Marriage Theorem states that there will be a set of musicians S such that the set of instruments they can collectively play, N(S), is smaller than the number of musicians:

$$|N(S)| < |S|. \quad (1)$$

In the case of this question the maximal matching is not perfect. Use the Ford Fulkerson algorithm to find a min cut which lets you find a set of musicians S for which equation (1) is true. In other words, a set of musicians which have a smaller set of instruments that they can collectively play.

1b) Find a maximal matching for the following bipartite graph:



2. Circulation with Demands

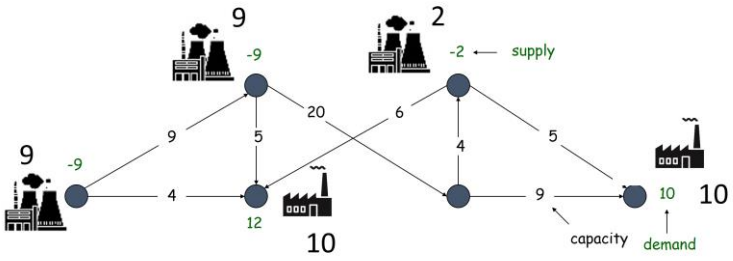
2a) The following is a flow network with supplies and demands on the nodes.

Each node is labelled with its supply (negative value) or demand (positive value)

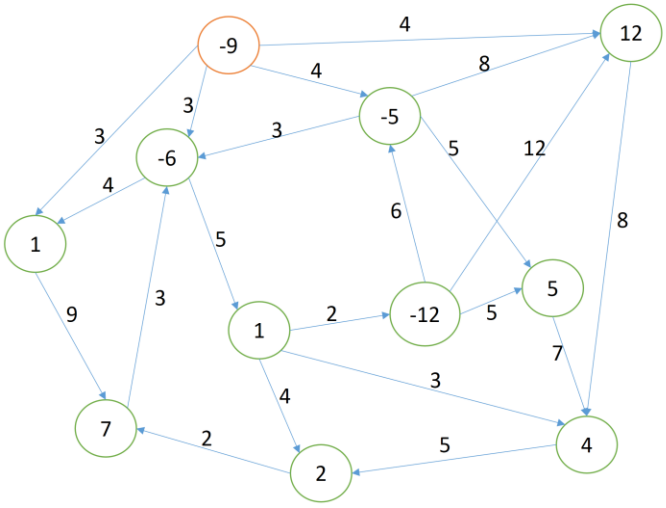
Each edge is labelled with its capacity.

Your task is to assign flows to the edges so that every node meets the conservation condition, that the sum of flows in minus the sum of flows out is equal to the demand on the node.

The flow on each edge cannot exceed its capacity.



2b) Now find a flow for the following flow network with supplies and demands:



You may also find the following link useful for additional background on circulation with demands:
https://github.com/SleekPanther/circulation-with-demands-network-flow